# A Numerical Method for Analyzing Electromagnetic Properties of a Moving Three-dimensional Object

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**The interaction of electromagnetic fields with a moving three-dimensional object is instructively meaningful to many practical applications. In this paper, the precise integration time domain method, combined with the parameters approximation technique, is used to numerically calculate the near-field components. Reliable results are obtained and the generality of the proposed method is revealed.**

***Index Terms*—Electromagnetic fields, near-field, numerical method, three-dimensional moving object.**

## I. Introduction

THERE are plentiful of researches dealing with the electromagnetic problem involving moving objects in which only 1-dimensional case and/or 2-dimensional case are considered. Here we present the trial to numerically analyze the electromagnetic interaction of a moving three-dimensional object. The precise integration time domain method (PITD) is employed owing to its efficiency and visualized results obtained directly in the time domain. Yee-grid is employed to arrange the electric\magnetic field components. Particularly, parameter approximation (PA) scheme is developed in the cells where the boundaries of the moving object reside. In those mixed-medium cells, the effective parameters are approximated via the weighted averaging procedure.Moreover, since the coefficient matrix is time-variant caused by the motion, an effective constant coefficient matrix in a single time interval is constructed. In the context of several numerical simulations, the effectiveness of the proposed method is confirmed. In addition, it is shown that the proposed method is general if only the proper coefficient matrix is developed

## II. Computational Model

The physic model considered in this paper is sketched in Fig. 1. In this digest, the fixed Yee-grid is of the size 5×5×10 cells. The object is a conductor and constantly moving along the +z-axis. The size of the conductor is 1×2×1. The excitation is a point-wise time-harmonic electric component source of 10MHz and unity amplitude and is placed at the bottom of the computation domain, as shown in Fig.1. It should be emphasized that a plane wave can also be introduced via the total-field-scattered-field technique. Since the major purpose is to observe the frequency shift caused by the movement of the object, the point-wise source is adequate. Perfectly conducting boundary condition which can be most conveniently implemented in the PITD method, is adopted in the borders of the computation domain.

## III. Parameter Approximation Scheme

The Yee cells near the boundaries include both the host medium and the moving medium. The scheme to determine the parameters in these cells is elaborated here.

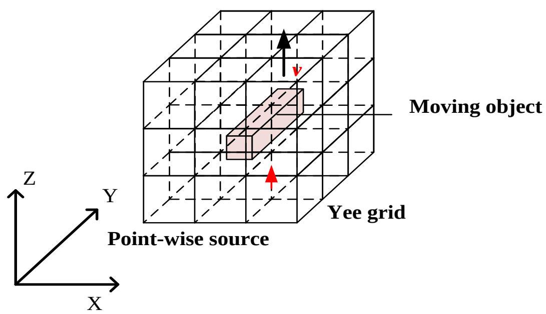


Fig. 1 Moving object in Yee-grid

Suppose that $\psi$ denotes the parameter of the cell, permittivity or permeability, for example. $\psi_h$ denotes that of the host medium and $\psi_m$ the moving medium, respectively. $f_{frac}$ is defined as the ratio of the area occupied by the moving medium to the area of the whole cell. Then $\psi$ is expressed as

$$\psi=\left(1-f_{frac}\right)\cdot\psi_h+f_{frac}\cdot\psi_m \tag{1}$$

## IV. Precise Integration Time domain

The PITD method is frequently used in analyzing electromagnetic wave problem [1]. In the PITD method, the Yee-grid inherited from the finite difference time domain (FDTD) method is employed to arrange the discrete electromagnetic field components in space. The electric components and the magnetic components appear alternatively by half spatial increment in each direction. On the contrary, the electric components and the magnetic components are sampled at the same series of time points. The modified discretized formulations of the Maxwell curl equations in the cells in which the moving material locates are given as a set of ordinary differential equations (ODEs).

$$\varepsilon\frac{\partial(E_x)}{\partial t}+\frac{\partial\varepsilon}{\partial t}E_x=\frac{H_z(j+1/2)-H_z(j-1/2)}{\Delta y}-\frac{H_y(k+1/2)-H_y(k-1/2)}{\Delta z} \tag{2}$$

$$\varepsilon\frac{\partial(E_y)}{\partial t}+\frac{\partial\varepsilon}{\partial t}E_y=\frac{H_x(k+1/2)-H_x(k-1/2)}{\Delta z}-\frac{H_z(i+1/2)-H_z(i-1/2)}{\Delta x} \tag{3}$$

$$\varepsilon\frac{\partial(E_z)}{\partial t}+\frac{\partial\varepsilon}{\partial t}E_z=\frac{H_y(i+1/2)-H_y(i-1/2)}{\Delta x}-\frac{H_x(j+1/2)-H_x(j-1/2)}{\Delta y} \tag{4}$$

$$-\mu\frac{\partial(H_x)}{\partial t}-\frac{\partial\mu}{\partial t}H_x=\frac{E_z(j+1)-E_z(j)}{\Delta y}-\frac{E_y(k+1)-E_y(k)}{\Delta z} \tag{5}$$

$$-\mu\frac{\partial(H_y)}{\partial t}-\frac{\partial\mu}{\partial t}H_y=\frac{E_x(k+1)-E_x(k)}{\Delta z}-\frac{E_z(i+1)-E_z(i)}{\Delta x} \tag{6}$$

$$-\mu\frac{\partial(H_z)}{\partial t}-\frac{\partial\mu}{\partial t}H_z=\frac{E_y(i+1)-E_y(i)}{\Delta x}-\frac{E_x(j+1)-E_x(j)}{\Delta y} \tag{7}$$

Note that the time partial differential terms of the permittivity and the permeability are introduced due to the movement. These two terms are related to the moving speed. Discretized formulations of the Maxwell curl equations in the other cells remain the same as conventional PITD form [1].

## V. PIECE-WISE CONSTANT COEFFICIENT MATRIX

The overall coefficient matrix **M** when the target is moving is varied with time.The formulation of the conventional PITD method has to be modified. Since **M** becomes time-variant, (2)~(7) are written in a general form as

$$\frac{d\mathbf{X}}{dt}=\mathbf{M}(t)\mathbf{X}+\mathbf{f}(t) \tag{8}$$

**X** consists of all the electromagnetic field components. The analytical solution of the set of ODEs is, in most cases, impossibly obtained. Fortunately, an effective time-constant substitution of time-variant **M**(*t*) can be constructed provided that the time step size Δ*t* is sufficiently small. The coefficient matrices designated as $\mathbf{M_s}$, $\mathbf{M_e}$, and $\mathbf{M_m}$ at the starting time point, the ending time point, and the middle time point, respectively, are linearly combined to construct the effective matrix **M** in a single time interval.The middle time point is totally included in the time interval, both the starting and the ending time points are joint points for two consecutive time intervals and equally shared by the two consecutive time intervals. As a result,the effective constant matrix **M** can be properly expressed as

$$\mathbf{M}=0.25\mathbf{M}_s+0.5\mathbf{M}_m+0.25\mathbf{M}_e \tag{9}$$

It is apparent that **M** over all time intervals becomes piece-wise constant, then the PITD method can be used to solve (8).

## VI. NUMERICAL RESULTS

In this section, the results of electromagnetic field tangled by the moving object under the settings in section II is presented. The waveform of $E_x$ in the backscatter direction and spectrum of $E_x$ are shown in Fig.2 and Fig. 3, respectively. It can be seen that the returned wave undergoes a Doppler frequency shift down to 39.2MHz and an amplitude decreasing which are in accordance with the theory [2]. Fig. 4(a) to Fig. 4(c) depict the spatial distribution of the electromagnetic field in different cross sections at three time points, providing a visualized results of the evolution of the electromagnetic field when a moving three-dimensional target is involved. i, j, and k denote the indices in *x*, *y*, and *z* directions respectively. It is clear that conversion point of the electromagnetic wave is dragged by the object along the moving direction.

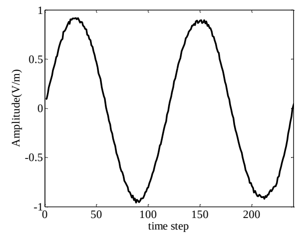


Fig. 2 Waveform of $E_x$ in the back-scatter direction

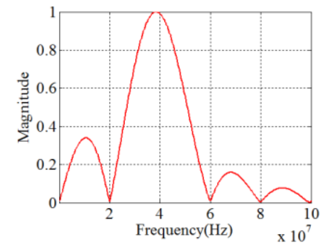


Fig. 3 Spectrum of the waveform in Fig. 2

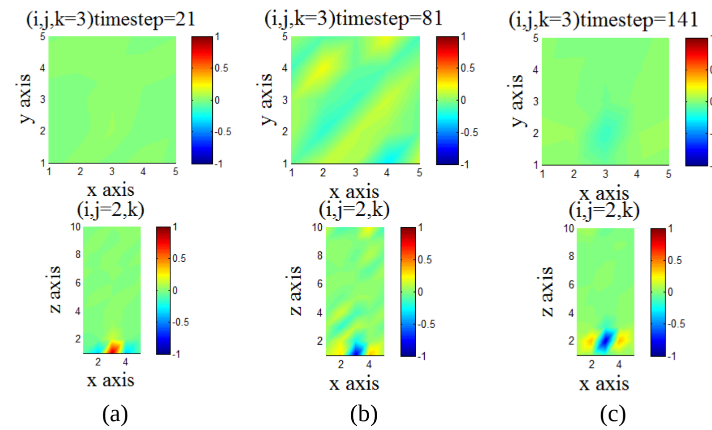


Fig. 4 Spatial distribution of $E_x$

## VII. CONCLUSION

The PA-PITD method is developed to the application involving three-dimensional object. Numerical results indicate that the anticipated Doppler frequency shift is observed for the time-harmonic excitation. This can be an evidence that the PA-PITD method is favorable to the problem considered here. Also, the PA-PITD method can be a strong tool to analyze the evolution of the electromagnetic field involving moving three-dimensional object. It should be addressed that the plane wave and the absorbing boundary condition can be introduced in the PA-PITD method [3]. Thus, we can discuss the Doppler phenomenon in an arbitrary azimuth angle. Moreover, the PA-PITD method is a general method in which multiple moving objects can be involved, both of which are elaborated in the full version of this paper.

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